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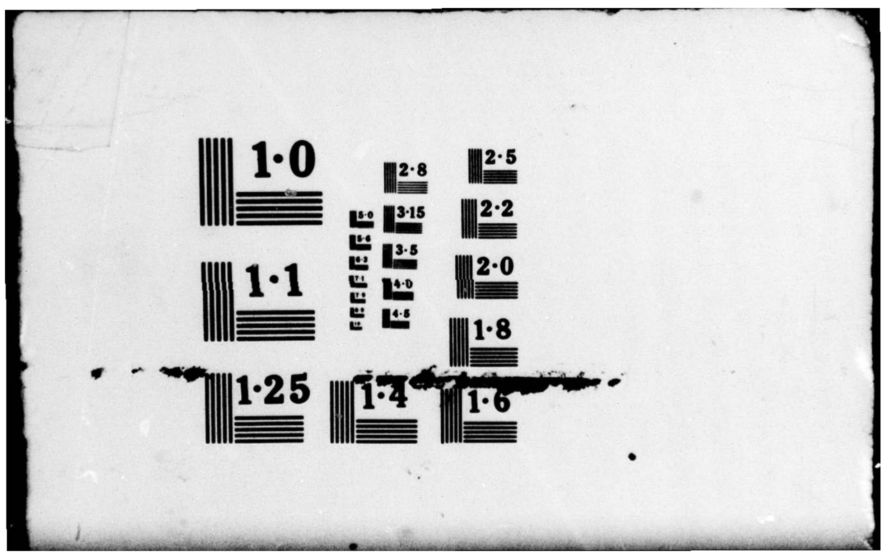
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OPTIMAL REDUCED-ORDER OBSERVERS FOR DISCRETE TIME-DELAYED SYSTEMS WITH UNCERTAIN OBSERVATIONS

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by

10 D.F. Liang

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RESUME

Ce rapport présente une généralisation de l'estimateur de Luenberger pour les systèmes stochastiques et déterministes discrets et à délai. Dans de tels systèmes, il est possible que les signaux détectés ne contiennent que du bruit et que seule la probabilité que ces cas surviennent est connue de l'estimateur. Cette généralisation contient les cas particuliers du filtre optimal de Kalman, du filtre de Nahi et de l'estimateur d'ordre réduit de Tse et Athans, et Leondes et Novak. L'utilisation de cette generalisation est illustrée par un exemple de poursuite radar. (NC)

ABSTRACT

This report presents the generalization of Luenberger's observer to discrete time-delayed deterministic and stochastic systems, when the detected signals may contain noise alone, and only the probability of occurrence of such cases is known to the estimator. The solution encompasses the special cases of Kalman's optimal filter, Nahi's filter, Tse and Athans and Leondes and Novak's reduced-order observers. The design of a reduced-order observer is illustrated using a radar-tracking problem. (U)

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1.0 INTRODUCTION

Many practical problems in navigation, weapon delivery and tracking systems require the use of the Kalman filtering technique to minimize the estimation errors of certain physical variables. In practical realization, however, the complexity of the optimal filtering technique may unduly burden the computer memory and computational capability. To circumvent this problem, there has been much interest in the construction of optimal and suboptimal reduced-order observers, along the line of Luenberger's observer [1]. One group of papers [2-3] essentially obtained the conditional mean estimate of the state, while minimizing the order of the filter. The other group [4-6] constructed the observer estimator by constraining the memory size of the observer. And they all made the classical assumption that the measurement signals received always contained the state variables.

In many realistic military applications, the presence of the state vector may be associated with a non-zero probability of false alarm due to fading or enemy interference. In the case that all the measurement outputs are corrupted by additive white noise, one can use the filtering algorithm of Nahi [7]. But to the author's knowledge, optimal and suboptimal observer-estimators have not been derived in the literature.

Therefore, this report is concerned with the generalization and extension of Luenberger's observer to discrete time-delayed deterministic and stochastic systems, when the signal detection process is associated with a non-zero probability of false alarm. The derivation makes use of the matrix minimum principle [10] to minimize the error-variance estimation criterion.

Section 2 presents an observer; its structural design can be considered as the extension of Aoki and Huddle's [4] fundamental structure in the domain of linear deterministic discrete time-delayed

systems under uncertain observations. Section 3 applies the matrix minimum principle to yield the minimum variance estimate of the state. This section considers the situation where some of the components of the measurement vector are noise-free, and the development is parallel to that of Yoshikawa and Kabayaski [8]. Section 4 presents a class of general reduced-order observers under uncertain observation, based upon the experience of Leondes and Novak [5,6]. The results can be shown to encompass the special cases of Kalman [9], Nahi [7] and Leondes and Novak [5,6]. A practical radar-tracking problem is presented for the purpose of illustration.

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2.0 DETERMINISTIC OBSERVER

Consider a general class of linear time-varying discrete systems involving multiple delays:

$$x(k+1) = \sum_{i=0}^{N} A_i(k)\, x\,(k-a_i) + \sum_{j=0}^{M} B_j\,(k)\, u\,(k-b_j) \tag{1}$$

where A_i and B_j are respectively time-varying n x n and n x p matrices, x is the n-dimensional state vector, and u is the p-dimensional control vector with $p \leq n$.

a_i and b_j are delay indices with $a_o = b_o = 0$, $a_i \geq a_{i-1}$, and $b_j \geq b_{j-1}$.

The output uncertain observation can be described by

$$y(k) = \gamma\,(k)\, C\,(k)\, x\,(k) \tag{2}$$

where the matrix C is m x c dimensional and $\gamma(k)$ is a binary quantity taking on values of 0 or 1 with the following mutually exclusive probability

$$Pr.[\gamma(k) = 1] = q(k) \tag{3}$$

and $$Pr.[\gamma(k) = 0] = 1 - q(k) \tag{4}$$

where $1-q(k)$ is the probability of occurrence of a false alarm, and

$$E\,[\gamma(k)\ \gamma(j)] = q(k)\ q(j),\ k\neq j \tag{5}$$

$$E\,[\gamma^2(k)] = q(k) \tag{6}$$

In some applications, the state variable x may not be directly available, or the dimension of the state model is too large that it may be desirable to reduce the order of the estimator. The general form of such an observer can be described by the relation

$$z(k+1) = \sum_{i=0}^{N} F_i(k)\ z\ (k-a_i) + \sum_{i=0}^{N} D_i(k)\ y\ (k-a_i) + \sum_{j=0}^{M} G_j(k)\ u\ (k-b_j) \tag{7}$$

and the estimate of x takes on the expression

$$\hat{x}(k) = P(k)\ z\ (k) + H(k)\ y\ (k) \tag{8}$$

where z is ℓ-dimensional and $n \geq \ell \geq n-m$. F_i, D_i, G_j, P and H are of the order $\ell \times \ell$, $\ell \times m$, $\ell \times p$, $n \times \ell$ and $n \times m$, respectively.

Remark 1: If there exists an $\ell \times n$ dimensional transformation matrix T such that

$$E_\gamma\,[z(k)] = E_\gamma\,[T\ (k)\ x\ (k)] \tag{9}$$

then, the necessary condition for $\hat{x}$ (k) to be an unbiased estimate of x (k) can be easily obtained as

$$P(k)\ T\ (k) + q(k)\ H\ (k)\ C\ (k) = I_n \tag{10}$$

Remark 2: Using the relation of Eqs. (7) and (9), one can obtain the Luenberger's necessary conditions

$$T(k+1) \sum_{i=0}^{N} A_i(k) = \sum_{i=0}^{N} F_i(k)\ T\ (k-a_i) + \sum_{i=0}^{N} q(k)\ D_i(k)\ C\ (k-a_i) \tag{11}$$

and

$$\sum_{j=0}^{M} G_j(k) = T\ (k+1) \sum_{j=0}^{M} B_j(k) \tag{12}$$

Theorem 1: Given a matrix T(k) which satisfies the relation of Eq. (9), then for the dynamic system of Eqs. (1) and (2), one can construct an ℓ-order observer in the form of Eqs. (7) and (8).

Proof: Let $F_i(k) = T(k+1)\ A_i(k)\ P\ (k-a_i)$ (13)

$$D_i(k) = T(k+1)\ A_i(k)\ H\ (k-a_i) \tag{14}$$

where P and H must be chosen to satisfy the constraining condition of Eq. (10).

Then, making use of Eqs. (1), (2), (7), (8) and (12) leads to

$$E_\gamma\ [z\ (k+1) - T\ (k+1)\ x\ (k+1)] = \sum_{i=0}^{N} T(k+1)\ A_i(k)$$

$$P(k-a_i)\ [z\ (k-a_i) - T\ (k-a_i)\ x\ (k-a_i)]$$

This implies that once the initial conditions are selected such that

$$z\ (-a_i) = T\ (-a_i)\ x\ (-a_i),\ i = 0, 1, 2, \ldots$$

then, $E_\gamma\ [z\ (k)] = E_\gamma[T\ (k)\ x\ (k)]$ for all $k = 0, 1, 2, \ldots$ and, hence, the existence of an ℓ-order observer.

3.0 OPTIMAL MINIMAL-ORDER OBSERVER FOR STOCHASTIC SYSTEMS

Consider a class of stochastic discrete time-delayed systems given by

$$x(k+1) = \sum_{i=0}^{N} A_i(k)\; x\,(k-a_i) + \sum_{j=0}^{M} B_j(k)\; u\,(k-b_j) + \xi(k) \tag{15}$$

and the uncertain observation is described by

$$y(k) = \gamma\,(k)\; C\,(k)\; x\,(k) + \eta\,(k) \tag{16}$$

where ξ and η represent independent zero-mean white-noise processes with known correlation matrices Q and R respectively. The binary sequence γ (k) is identical to that of Section 2.

The initial states x (0) and x $(-a_i)$, for i=0, 1, ..., N are independent zero-mean Gaussian random vectors with a positive definite covariance matrix

$E_x\, E_\gamma\, \{x\,(-a_i)\; x^T\,(-a_k)\} = V_o\,(a_i,\, a_k)$ for i = 0, 1, ..., N and k = 0, 1, ..., j.

Theorem 2: Given the stochastic system of Eqs. (15) and (16) with the observer constructed in the form of Eqs. (7) and (8), the error variance equation is described by

$$V_{\tilde{x}}\,(k+1) = [I_n - q\,(k+1)\; H\,(k+1)\; C\,(k+1)]\; [\sum_{i,j=0}^{N} A_i\,(k)\; V_{\tilde{x}}\,(k-a_i,\; k-a_j)\; A_j^{\;T}\,(k) + Q\,(k)] \tag{17}$$

where the optimal value of H (k+1) minimizing the error-variance cost function

$$J = E_x\, E_\gamma\; \{\; \tilde{x}^T\,(k+1)\; M\,(k+1)\; \tilde{x}\,(k+1)\} \tag{18}$$

is given by

$$H(k+1) = [\;\{\sum_{i,j=0}^{N} A_i\,(k)\; V_{\tilde{x}}\,(k-a_i,\; k-a_j)\; A_j^{\;T}(k) + Q\,(k)\}\; C^T\,(k+1)\; q\,(k+1)]\; [R\,(k+1) + q^2\,(k+1)\; C\,(k+1)$$

$$\{ \sum_{i,j=0}^{N} A_i (k) V_{\tilde{x}} (k-a_i, k-a_j) A_j^T (k) + Q(k)\} C^T (k+1)$$

$$+ \{ q (k+1) - q^2 (k+1) \} C (k+1) V_x (k+1) C^T (k+1)]^{-1} \quad (19)$$

where

$$V_x(k+1) = V_{\tilde{x}} (k+1) + \sum_{i,j=0}^{N} A_i(k) \hat{x} (k-a_i) \hat{x}^T(k-a_j) A_j^T (k).$$

Proof: Since $\tilde{x} (k+1) = x (k+1) - \hat{x} (k+1)$, utilizing the expression of Eqs. (7) and (8) yields

$$\tilde{x}(k+1) = [I_n - H (k+1) q (k+1) C (k+1)] [\sum_{i=0}^{N} A_i (k)$$

$$\tilde{x} (k-a_i) + \xi (k)] + H (k+1) [q (k+1) - \gamma (k+1)]$$

$$C (k+1) x (k+1) + H (k+1) \eta (k+1) \quad (20)$$

The error-variance equation is now obtainable by taking the expectation of Eq. (20) multiplied by its own transpose. The optimal value of H (k+1) can be derived by setting the gradient of Eq. (18) equal to the null matrix [10]. The result is simply that of Eq. (19), which in turn reduces the error-variance equation to Eq. (17).

Remark 3: Substitution of Eqs. (7), (12) - (14) into Eq. (8) also yields

$$\hat{x}(k+1) = [I_n - q(k+1) H(k+1) C (k+1)] [\sum_{i=0}^{N} A_i (k) \hat{x} (k-a_i)$$

$$+ \sum_{j=0}^{M} B_j (k) u (k-b_j)] + H (k+1) y (k+1) \quad (21)$$

Notice that the estimator of Eq. (21) together with Eqs. (17) and (19) constitute Nahi's estimator [7] for a linear discrete system with uncertain observations. In the special case that q(k) = 1, the above algorithms reduce to Kalman's optimal filter. It may also be noted that the optimum value of H(k+1) and the error-variance equation of Eq. (17) are entirely independent of the control vector u.

On the other hand, the output vector $z(k+1)$ of Eq. (9) is an estimate of $T(k+1)\ x\ (k+1)$ with an observer error $\varepsilon(k+1)$ that can be easily found to satisfy the equation

$$\varepsilon(k+1) = \sum_{i=0}^{N} [F_i(k)\{z(k-a_i) - T(k-a_i)\,x(k-a_i)\} + D_i(k)\,\eta(k-a_i)$$

$$+ D_i(k)\{\gamma(k-a_i) - q(k-a_i)\}\,C(k-a_i)\,x(k-a_i)]$$

$$- T(k+1)\,\xi(k). \tag{22}$$

and the error-variance equation of $\varepsilon(k+1)$ is given as

$$V_\varepsilon(k+1) = T(k+1)\,[\sum_{i,j=0}^{N} A_i(k)\,P(k-a_i)\,V_\varepsilon(k-a_i, k-a_j)$$

$$P^T(k-a_j)\,A_j^T(k) + A_i(k)\,H(k-a_i)\,R(k-a_i, k-a_j)$$

$$H^T(k-a_j)\,A_j^T(k) + Q(k)]\,T^T(k+1)$$

$$+ \sum_{i=0}^{N} T(k+1)\{q(k-a_i) - q^2(k-a_i)\}\,A_i(k)\,H(k-a_i)$$

$$C(k-a_i)\,V_x(k-a_i)\,C^T(k-a_i)\,H^T(k-a_i)\,A_i^T(k)\,T^T(k+1) \tag{23}$$

Now the design of the observer is to be concentrated on the selection of the matrices P and T, satisfying the constraint of Eq. (10).

In this section, the observation equation of Eq. (16) is assumed to contain m_1 dimensional noise-free components, namely

$$y(k) = \gamma(k)\begin{bmatrix} C_1(k) \\ C_2(k) \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ \eta_1(k) \end{bmatrix} \tag{24}$$

where C_1 and C_2 are respectively, $m_1 \times n$ and $m-m_1 \times n$ dimensional, with noise correlation matrix

$$R_1(k) = \begin{bmatrix} 0_{m_1\,m_1} & 0 \\ 0 & R_1'(k) \end{bmatrix}$$

where $R_1'(k) > 0$. For this case, it can be easily shown that the dimension of the optimal minimal-order observer is $n-m_1$ [3].

Now, partition the matrix C(k), H(k) and the estimator $\hat{x}(k)$ in the forms of

$$C(k) = \begin{bmatrix} C_{11}(k) & C_{12}(k) \\ C_{21}(k) & C_{22}(k) \end{bmatrix}$$

$$H(k) = \begin{bmatrix} H_1\ (k) \\ H_2\ (k) \end{bmatrix}$$

and

$$\hat{x}(k) = \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} \tag{25}$$

Using the compatibility theorem of Tse and Athans [3] and following the procedure similar to that of Yoshikawa and Kabayaski [8], one obtains

$$q(k)\ H_1(k) = C_{11}{}^{-1}\ (k)\ \{[I_{m_1}|0] - q(k)\ C_{12}(k)\ H_2\ (k)\} \tag{26}$$

and after some algebraic manipulations, one can show that the pair

$$P(k) = \begin{bmatrix} -C_{11}{}^{-1}(k)C_{12}(k) \\ I_{n-m_1} \end{bmatrix} \tag{27}$$

and

$$T(k) = \left[-q(k)\ H_2(k)\ C_1\ (k) \,\middle|\, I_{n-m_1} - q(k)\ H_2(k)\ C_2(k) \right] \tag{28}$$

specifies one optimal minimal-order observer, and $H_1(k)$ is uniquely specified in terms of $H_2(k)$, if $q(k) \neq 0$.

4.0 GENERAL REDUCED-ORDER OBSERVERS

This section considers a general class of measurement models

$$y(k) = \gamma(k) \begin{bmatrix} I & 0 \\ C_1(k) & C_2(k) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} \eta_1(k) \\ \eta_2(k) \end{bmatrix} \tag{29}$$

where x_1 is m_1 dimensional and x_2 is $n-m_1$ dimensional, since in many physical systems some of the measurement outputs are simply the

corrupted signals of the states involved. And it is further assumed that m_1 components of the measurement vector are relatively noise-free compared to the remaining outputs. The problem here is to construct a reduced-order observer of $n-m_1$ dimensional using these m_1 measurement outputs.

It could easily be shown that in the case of $\gamma(k) = 1$, $N = 0$, $C_1 = 0$, Eqs. (15) and (29) constitute the canonical class of systems considered by Leondes and Novak [6]. When $\gamma(k) = 1$, $N = 0$, $m_1 = m$, the solution of the problem is reduced to that of Leondes and Novak [5] If one were to set $\gamma(k) = 1$, $m_1 = 0$, $N = 0$, the problem becomes the special case of Kalman filter [9], and when $m_1 = 0$, $N = 0$, the solution is equivalent to that of Nahi [7].

Therefore, the modeling of Eqs. (15) and (29) does not cover only the reduced-order observer theory, when all measurement signals obtained are assumed to contain the state variables. It also presents a rather broad spectrum of reduced-order observers, when the detected signals are associated with the probability of false alarm.

Based upon the experience of Leondes and Novak [5], [6] in constructing a classical form of reduced-order observers satisfying the constraining relation of Eq. (10), one can set

$$P = \begin{bmatrix} 0 \\ I_{n-m_1} \end{bmatrix} \tag{30}$$

$$H = \begin{bmatrix} q^{-1}(k)\, I_{m_1} & 0 \\ K_1 & K_2 \end{bmatrix} \tag{31}$$

From Eq. (10), it can be shown that

$$T(k) = [-q(k)\,\{K_1 + K_2\, C_1\} \mid I_{n-m_1} - q(k)\, K_2\, C_2] \tag{32}$$

Application of Eq. (16) and the definition of $\varepsilon(k)$ brings about

$$V_{\tilde{x}}(k) = [q(k) - q^2(k)]\ H(k)\ C(k)\ V_x(k)\ C^T(k)\ H^T(k)$$
$$+ [P(k)\ H(k)] \begin{bmatrix} V_\varepsilon(k) & 0 \\ 0 & R(k) \end{bmatrix} \begin{bmatrix} P(k) \\ H(k) \end{bmatrix} \tag{33}$$

Partition Eq. (23) in the form of

$$V_\varepsilon(k+1) = T(k+1) \begin{bmatrix} W_{11}(k) & W_{12}(k) \\ W_{21}(k) & W_{22}(k) \end{bmatrix} T^T(k+1) \tag{34}$$

then, the trace of Eq. (33) can be derived using Eqs. (30) - (32). From the relations of Eq. (7), (22), and the definitions of $\varepsilon(k+1)$ and $W(k)$, one can show that

$$V_x(k+1) = W(k) + \sum_{i,j=0}^{N} A_i(k)\ \hat{x}(k-a_i)\ \hat{x}^T(k-a_j)\ A_j^T(k)$$

Finally, K_1 and K_2 are optimized using the matrix minimum principle; the results are:

$$[K_{11}(k+1)\ K_2(k+1)] = \left[q(k+1)W_{12}(k) \,|\, q(k+1)\{W_{12}(k)\ C_1^T(k+1) + W_{22}(k)C_2^T(k+1)\} \right]$$

$$\begin{bmatrix} R_{11}(k+1) + q^2(k+1)W_{11}(k) + \{q(k+1) - q^2(k+1)\}\ V_{x_{11}}(k+1) \\ R_{12}(k+1) + q^2(k+1)\ \{ C_1(k+1)W_{11}(k) + C_2(k+1)W_{12}(k) \} \\ + \{ q(k+1) - q^2(k+1) \}\ \{ C_2(k+1)V_{x_{12}}(k+1) + C_1(k+1)\ V_{x_{11}}(k+1) \} \end{bmatrix} \tag{35}$$

$$\left. \begin{matrix} R_{12}(k+1) + q^2(k+1)\ \{ W_{11}(k)\ C_1^T + W_{12}(k)\ C_2^T(k+1) \} \\ + \{ q(k+1) - q^2(k+1) \}\ \{ V_{x_{12}}(k+1)\ C_2^T(k+1) + V_{x_{11}}(k+1)\ C_1^T(k+1) \} \\ R_{22}(k+1) + q^2(k+1)\ C_1(k+1)\ \{ W_{11}(k)\ C_1^T(k+1) + W_{12}(k) \\ C_2^T(k+)\ \} + q^2(k+1)\ C_2(k+1)\ W_{22}(k)\ C_2^T(k+1) \\ + \{ q(k+1) - q^2(k+1) \}\ \{ C_2(k+1)\ V_{x_{22}}(k+1)\ C_2^T(k+1) \\ + C_1(k+1)\ V_{x_{11}}(k+1)\ C_1^T(k+1) + 2\ C_1(k+1)\ V_{x_{12}}(k+1)\ C_2^T(k+1) \} \end{matrix} \right]^{-1}$$

where the noise covariance matrix of (v_1, v_2) and the covariance of $x(k+1)$ have been respectively partitioned as

$$R(k+1) = \begin{bmatrix} R_{11}(k+1) & R_{12}(k+1) \\ R_{21}(k+1) & R_{22}(k+1) \end{bmatrix}$$

and

$$V_x(k+1) = \begin{bmatrix} V_{x_{11}}(k+1) & V_{x_{12}}(k+1) \\ V_{x_{21}}(k+1) & V_{x_{22}}(k+1) \end{bmatrix}$$

In order to guarantee a unique solution for K_1 and K_2, it is sufficient that Q_{11} and R_{22} be positive definite.

The observer can be initialized by defining

$$W(0) = \sum_{i,j=0}^{N} A_i(0)\, V_0\, (a_i, a_j)\, A_j^T(0) + Q_0$$

then K_1, K_2, V_ε, W and $V_{\tilde{x}}$ can sequentially be evaluated using Eqs. (23), (33) - (35).

5.0 ILLUSTRATIVE EXAMPLE OF A RADAR-TRACKING PROBLEM

In order to illustrate the performance of the proposed reduced-order observer, the radar-tracking system of Leondes and Novak [6] is considered:

$$\begin{bmatrix} x(k+1) \\ \dot{x}(k+1) \\ \ddot{x}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ \dot{x}(k) \\ \ddot{x}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} W(k)$$

The measurement model is given by

$$y(k) = \gamma(k) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ \dot{x}(k) \\ \ddot{x}(k) \end{bmatrix} + v(k)$$

where $\gamma(k)$ is a binary sequence of 0 or 1 with probability Prob. $[\gamma(k) = 1] = p$.

In the numerical simulation, three different values of p were selected. When p = 1, the Kalman filter estimates of a 30-run average and the estimates of the reduced-order observer are compared to the actual state values. When p = 0.8 and 0.6, the estimates of the reduced-order observers are compared to those of Nahi estimates. From the results of Figs. 1-5, it can be seen that estimates obtained from the reduced-order observers are slightly less accurate than those of Kalman or Nahi estimates. As expected, the lowest error variances occur when there is a zero probability of false alarm.

6.0 CONCLUSIONS

This paper has extended the Luenberger's observer to deterministic and stochastic discrete time-delayed systems, when the detector decision processess are associated with a non zero probability of false alarm. The derivation makes use of the matrix minimum principle to minimize the error-variance cost function. It can be easily shown that the reduced-order observers of Leondes and Novak [5], [6], Kalman [9] as well as Nahi [7] estimates may be considered as special cases of the proposed reduced-order observer solution. The result is illustrated by a radar-tracking problem.

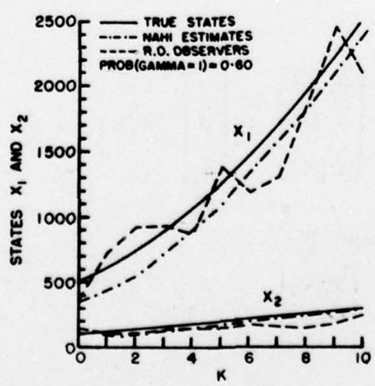

FIGURE I

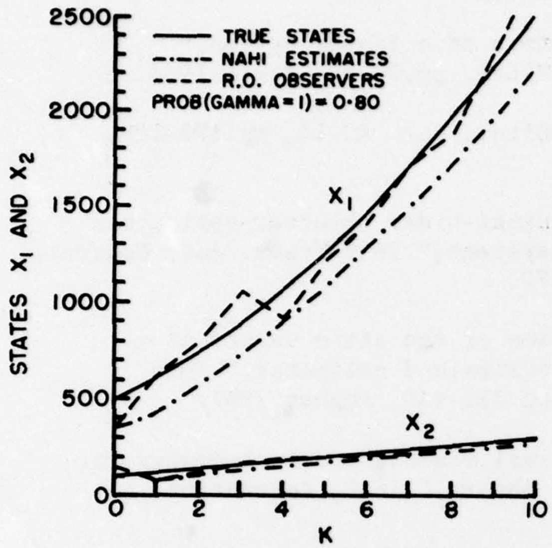

FIGURE 2

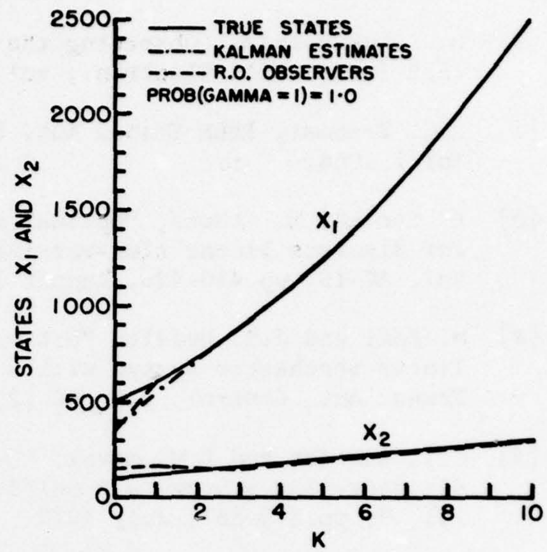

FIGURE 3

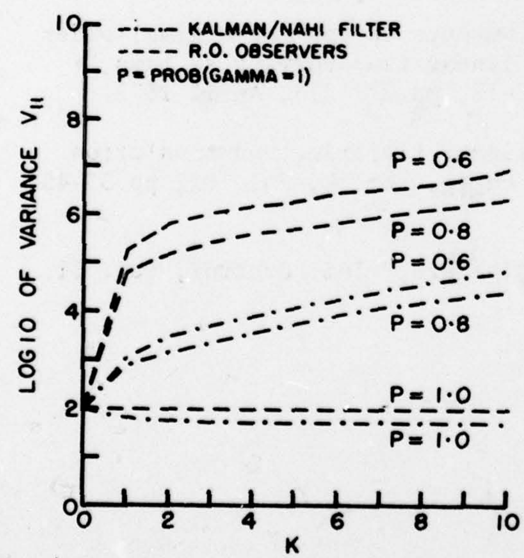

FIGURE 4

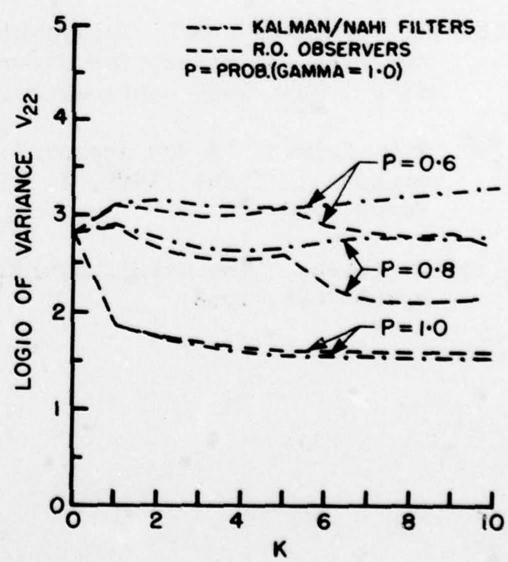

FIGURE 5

7.0 REFERENCES

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APPENDIX

A. Nahi's filter ($m_1 = 0$)

In this case, the observer state vector z is n-dimensional,

$K_1 = 0, \quad K_2 = K, \quad H = K, \quad P = I_n$

and $T(k) = I_n - q(k)\, K(k)\, C(k)$.

Substitution of Eqs. (7) and (10) into (8) yields

$$\hat{x}(k+1) = \sum_{i=0}^{N} A_i(k)\, \hat{x}(k-a_i) + K(k+1) \{ y(k+1) - q(k+1)\, C(k+1) \sum_{i=0}^{N} A_i(k)\, \hat{x}(k-a_i) \} \quad (36)$$

and from Eq. (34)

$$V_{\tilde{x}}(k+1) = [q(k+1) - q^2(k+1)]\, K(k+1)\, C(k+1)\, V_x(k+1)\, C^T(k+1)\, K^T(k+1) + V_\varepsilon(k+1) + K(k+1)\, R(k+1)\, K(k+1) \quad (37)$$

One also has the relation

$$W(k) = \sum_{i,j=0}^{N} A_i(k) [V_\varepsilon(k-a_i, k-a_j) + K(k-a_i)\, R(k-a_i, k-a_j)\, K^T(k-a_j) + \{ q(k-a_i) - q^2(k-a_i) \}\, K(k-a_i)\, C(k-a_i)\, V_x(k-a_i)\, C^T(k-a_j)\, K^T(k-a_j)\, \delta_{k-a_i, k-a_j}]\, A_j^T(k) + Q(k)$$
$$= \sum_{i,j=0}^{N} A_i(k)\, V_{\tilde{x}}(k-a_i, k-a_j)\, A_j^T(k) + Q(k)$$
$$= V_{\tilde{x}}(k+1|k) \quad (38)$$

and Eq. (35) becomes

$$K(k+1) = q(k+1)\, V_{\tilde{x}}(k+1|k)\, C^{T}(k+1)\, \Big[R(k+1) + q^2(k+1)\, C(k+1)\, V_{\tilde{x}}(k+1|k)\, C^{T}(k+1) + \{ q(k+1) - q^2(k+1) \}\, C(k+1)\, V_x(k+1)\, C^{T}(k+1) \Big]^{-1} \tag{39}$$

Finally, application of Eqs. (33), (37) and (39) leads to

$$V_{\tilde{x}}(k+1) = [I - q(k+1)\, K(k+1)\, C(k+1)]\, V_{\tilde{x}}(k+1|k) \tag{40}$$

It is now obvious that Eqs. (36) to (40) constitute the Nahi type filter under uncertain observations, and in the special case of $\gamma(k) = p(k) = 1$, they are reduced to Kalman filter.

B. The equivalence of the results presented in this paper and those of Leondes and Novak [5], [6] can easily be established using Eqs. (23), (30) - (33) and (35).

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